ANSWERS

1	c) A = {x: x = n^3 + n, n ∈ W, n < 5}	10	b) $4i$
2	c) $A^l \cap B^l$	11	a) 3024
3	a) $x = \frac{3}{4}$, $y = \frac{33}{4}$	12	(d) {1, 2, 3}
4	d) 2^{mn-1}	13	a) x ∈ (10, ∞)
5	b) $\frac{2\pi}{3}$	14	c) [0, ∞)
6	c) 0	15	d) {(-1, 1), (1, 1), (2, 4)}
7	a) 0	16	c) 120
8	b) 0	17	d) 64
9	c) { }	18	d) $a^2 + b^2 = c^2 + d^2$
19	(A) Both A and R are true and R is the correct explanation of A		
20	(B) Both A and R are true but R is NOT the correct explanation of A		

21

$\cos 2\theta \cdot \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$= \cos 2\theta \cos 2\phi + \sin(\theta - \phi + \theta + \phi)\sin(\theta - \phi - \theta - \phi)$

$= \cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi$

$= \cos(2\theta + 2\phi) = \cos 2(\theta + \phi)$

- OR -

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} \Rightarrow \cos x = \pm\frac{4}{5}$$

x lies in 2nd quadrant, $\cos x$ is (–ve). $\therefore \cos x = \frac{4}{5}$

$$\sin^2 y = 1 - \cos^2 y = 1 - \left(\frac{-12}{13}\right)^2 = \frac{25}{169} \Rightarrow \sin y = \pm\frac{5}{13}$$

Since, y lies in 2nd quadrant, $\sin y$ is (+ve) $\therefore \sin y = \frac{5}{13}$

$\sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

$$= \frac{3}{5}\times\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\times\frac{5}{13} = \frac{-36}{65} - \frac{20}{65} = \frac{-56}{65}$$

22

$z_1 \cdot z_2 = (1 - i)(-2 + 4i) = 2 + 6i$

$$\frac{2+6i}{1+i} \cdot \frac{1-i}{1-i} = \frac{(2+6i)(1-i)}{(1+i)(1-i)} = 4 + 2i$$

Imaginary part = 2

23

$30 < \frac{5}{9} \times (F - 32) < 35$,

$\frac{9}{5} \times (30) < (F - 32) < \frac{9}{5} \times 35$

$54 < (F - 32) < 63$

$86 < F < 95$

24

$$\frac{9!}{4!} + 5.\frac{9!}{5!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow \frac{9!}{4!} + \frac{9!}{4!} = \frac{10!}{(10-r)!} \quad \Rightarrow 2.\frac{9!}{4!} = \frac{10.9!}{(10-r)!}$$

$\Rightarrow (10 - r)! = 5! \quad \Rightarrow r = 5$

- OR -

Total words starting with A are 4! = 24

Total words starting with G are $\frac{4!}{2!} = 12$

Total words starting with I are $\frac{4!}{2!} = 12$

49th, word starting with N is NAAGI

50th, word starting with N is NAAIG.

25	$\text{Mean} = \frac{72}{8} = 9$ $3+2+1+3+4+5+1+3 = 22$ Mean Deviation about Mean $= \frac{22}{8} = 2.75$
26	(i) $A - B = \{2, 3, 5, 7\} - \{1, 2, 3, 4, 6, 8\} = \{5, 7\}$ $A \cap B^C = \{2, 3, 5, 7\} \cap \{5, 7, 9, 10\} = \{5, 7\}$ $A - B = A \cap B^C$ (ii) $A \cup B = \{2, 3, 5, 7\} \cup \{1, 2, 3, 4, 6, 8\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $(A \cup B)^C = U - (A \cup B) = \{9, 10\}$ $A^C \cap B^C = \{1, 4, 6, 8, 9, 10\} \cap \{5, 7, 9, 10\} = \{9, 10\}$ $(A \cup B)^C = A^C \cap B^C$
28	(i) $28 = x^2 + 3 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$ (ii) R={(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7),(16,8),(18,9),(20,10)} Domain = {2,4,6,8,10,12,14,16,18,20} Range = {1,2,3,4,5,6,7,8,9,10} $f(x) = x^2 - 3x + 1$ $f(2x) = f(x)$ $f(2x) = (2x)^2 - 3(2x) + 1 = 4x^2 - 6x + 1$ $4x^2 - 6x + 1 = x^2 - 3x + 1$ $3x^2 - 3x = 0$ $x = 0$ or $x = 1$ (i) $f + g = 1 + 0 = 1$ & $f + g = -1 + 1 = 0$ (ii) $\frac{f}{g} = \frac{1}{0}$ = Not defined & $\frac{f}{g} = \frac{-1}{1} = -1$
27	L.H.S. $= \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x} = \frac{2\sin 3x \cos 2x - 2\sin 3x}{-2\sin 3x \sin 2x} = -\frac{\sin 3x(\cos 2x - 1)}{\sin 3x \sin 2x}$ $= \frac{1 - \cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x \cos x} = \tan x$ = R.H.S. - OR - Given, L.H.S.= $\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$ $= \sin x \cos x [\tan x + \cot x]$ $= \sin x \cos x \left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right]$ $= \sin x \cos x \left[\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right]$ $= \sin^2 x + \cos^2 x$ $= 1$
29	Let $z = \frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{6+9i-4i+6}{2-i+4i+2}$ $= \frac{12+5i}{4+3i} = \left(\frac{12+5i}{4+3i}\right) \times \left(\frac{4-3i}{4-3i}\right)$ $= \frac{48-36i+20i+15}{4^2-(3i)^2} = \frac{63-16i}{16+9} = \frac{63}{25} - \frac{16}{25}i$ $\therefore$ Conjugate of $z = \frac{63}{25} + \frac{16}{25}i$ -Or- Given, $(x+iy)^3 = u + iv$ $\Rightarrow x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u + iv$ $\Rightarrow x^3 - iy^3 + 3x^2yi - 3xy^2 = u + iv$ $\Rightarrow (x^3 - 3xy^2) + i(3x^2y - y^3) = u + iv$ Comparing the equating real and imaginary parts, $\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$ $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$
30	$\frac{12!}{3! \cdot 2! \cdot 2!} = \frac{479001600}{6 \cdot 2 \cdot 2} = \frac{479001600}{24} = 19958400$ $\frac{11!}{3! \cdot 2! \cdot 2!} = \frac{39916800}{24} = 1663200$ $\frac{7!}{2!} = \frac{5040}{2} = 2520$ $\frac{6!}{3! \cdot 2!} = \frac{720}{6 \cdot 2} = \frac{720}{12} = 60$ $2520 \times 60 = 151200$

31		

x_i	f_i	$c.f$
15	3	3
21	5	8
27	6	14
30	7	21
35	8	29

$\therefore$ Median = 30

$\lvert x_i - M\rvert$	15	9	3	0	5
f_i	3	5	6	7	8
$f_i\lvert x_i - M\rvert$	45	45	18	0	40

$$\sum_{i=1}^{5} f_i = 29, \sum_{i=1}^{5} f_j\,|x_i - M| = 148$$

$$\therefore \text{M.D. (M)} = \frac{1}{N}\sum_{i=1}^{5} f_i\,|x_i - M| = \frac{1}{29}\times 148 = 5.1$$

32

$n(U) = 100$

$n(A) = 60$

$n(B) = 48$

$n(A\cap B) = 22$

$n(A\cap C) = 30$

And: $C \subseteq A$

(i) $n(B\cup C) = 48 + 30 - 0 = 78 \Rightarrow n(B'\cap C') = 100 - 78 = \boxed{22}$

$n(B'\cap C') = 22$

(ii) $n(A\cup B) = 60 + 48 - 22 = \boxed{86}$

$n(A\cup B) = 86$

33

Given, L.H.S.= $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)$

$[\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos(\frac{x-y}{2})]$

$\Rightarrow 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\cos\left(-\frac{\pi}{13}\right)$

$\Rightarrow 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\cos\left(\frac{\pi}{13}\right)$

$\Rightarrow 2\cos\frac{\pi}{13}\left(\cos\frac{9\pi}{13} + \cos\left(\frac{4\pi}{13}\right)\right)$

$= 2\cos\frac{\pi}{13}\left(2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right)$

$= 4\cos\frac{\pi}{13}\left(\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{5\pi}{26}\right)\right)$

$= 4\cos\frac{\pi}{13}\left(0\times\cos\left(\frac{5\pi}{26}\right)\right)$

$= 0$

- OR -

$\pi < x < \frac{3\pi}{2}$, $\cos x$ is negative. Also $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$.

$\sin\frac{x}{2}$ is positive and $\cos\frac{x}{2}$ is negative.

$\sec^2 x = 1 + \tan^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

$\cos^2 x = \frac{16}{25}$ or $\cos x = -\frac{4}{5}$

$2\sin^2\frac{x}{2} = 1 - \cos x = 1 + \frac{4}{5} = \frac{9}{5}$ $\quad \sin^2\frac{x}{2} = \frac{9}{10}$

$\sin\frac{x}{2} = \frac{3}{\sqrt{10}}$

$2\cos^2\frac{x}{2} = 1 + \cos x = 1 - \frac{4}{5} = \frac{1}{5}$ $\quad \cos^2\frac{x}{2} = \frac{1}{10}$

$\cos\frac{x}{2} = -\frac{1}{\sqrt{10}}$

34

$5(2x-7) - 3(2x+3) \le 0$

$\Rightarrow (10x - 35) - (6x + 9) \le 0$

$\Rightarrow 4x - 44 \le 0$

$\Rightarrow 4x \le 44$

- OR -
-

$\Rightarrow x \leq 11$

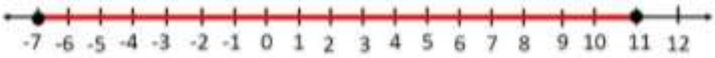
Also, $2x + 19 \leq 6x + 47$

$\Rightarrow 19 - 47 \leq 6x - 2x$

$\Rightarrow -28 \leq 4x$

$\Rightarrow -7 \leq x$

From (1) & (2), the solution is $[-7, 11]$



According to question we have, $x > 5$………………(1)

Also, the sum of the two integers is less than 23.

$\therefore x + (x + 2) < 23$

$\Rightarrow 2x + 2 < 23$

$\Rightarrow 2x < 21$

$\Rightarrow x < 10.5$ …………(2)

From equation (1) and (2), we get $5 < x < 10.5$.

The possible values of x are 6, 8 and 10.

The required possible pairs are (6, 8), (8, 10) and (10, 12).

35

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 105}{30}$	y_i^2	$f_i y_i$	$f_i y_i 2$
0-30	2	15	−3	9	−6	18
30-60	3	45	−2	4	−6	12
60-90	5	75	−1	1	−5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h$

$= 105 + \frac{2}{30} \times 30 = 105 + 2 = 107$

$Variance(\sigma^2) = \frac{h^2}{N^2}\left[N \sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$

$= \frac{(30)^2}{(30)^2}[30 \times 76 - (2)^2]$

$= 2280 - 4$

$= 2276$

SD = √2276 = 47.7

36

(i) Domain: $[-10, 10]$

Range: $[0, 10]$

(ii) Domain : $x \in \mathbb{R},\ x \neq -2,\ x \neq 3$ $\quad (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

Range : $(-\infty, \infty)$

37

(i) n(Only Music) = n(M) − n(M∩D) = 55 – 20 = 35

(ii) n(M ∪ D) = 55 + 45 – 20 = 80

(iii) (a) n(Only Dance) = 25

n(Neither) = 100 − n(M∪D) = 100 – 80 = 20

An: 25 + 20 = 45

38

(i) The digits which can be used are, 6, 5, 4, 2, 1

Number of ways to fill the 5 places = 5! = 120

(ii) Digits which can be used in AC code are 7, 8, 9

Total AC code = ${}^3P_2$ = 3! = 6

(iii) If AC and PC are fixed then only 5 digits is to be filled if digits can repeat then total ways = 10^5

- OR -

Total ways = 5 × 5 × 5 × 5 × 5 = 3125